Daniel Waterman, Department of Mathematics
Syracuse University, Syracuse, New York 13210

NEW RESULTS ON FUNCTION CLASSES INVARIANT
UNDER CHANGE OF VARIABLE

Let X be a class of real functions with domain D, either an interval or the circle group. We say that X is <u>invariant under change of variable</u> if $f \in X$ implies $f_0 g \in X$ for every $g : D \to D$ homeomorphically. There are two principal ways in which such classes arise:

(i) A class of functions may be defined by an intrinsic property of the functions, such as a condition on the corresponding interval functions (if $I = (a,b)$, $f(I) = f(b) - f(a)$) which is independent of the length of intervals, or by invariant properties of the functions' level sets, such as a condition on the Banach indicatrix.

(ii) Given a property P, we may consider the class of functions f such that $f \circ g$ has property P for all admissible g.

Examples of the first type come readily to mind. The functions of bounded variation and most classes of functions of generalized bounded variation are of this type as is the class of functions f such that $\varphi(n(y;f)) \in L^p$, where φ is an increasing non-negative function on R^+ and $n(y;f)$ is the Banach indicatrix of f. Examples of the second type are easily produced. Consider those f such that

146