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On Strong Essential Cluster Sets

1. Let H, R and M* stand for the open upper half plane, real line and Lebesgue outer measure, respectively. M* is linear or planar; the choice will be clear from the context. Let L(x) denote the ray in H emanating from $x \in R$ in the direction $\pi/2$ and let L(x,r) be a segment of L(x) with one end at x and of length r.

Let {I} be the collection of closed rectangles of the form $[a,b]\times[0,k]$, $a<0<b$, a, b and k are rationals. For $I \in \{I\}$ let $I(x_0)$ denote the closed rectangle obtained by mapping (x,y) into $(x_0 + x, y)$. The strong outer upper density of a set $E \subset H$ at x is defined by

$$d^*_s(E,x) = \lim_{n\to\infty}\left[\sup_{D(I)<1/n}\left\{\frac{M^*(I(x)\ E)}{M^*(I(x))} : I \in \{I\}\right\}\right]$$

where D(I) denotes the diameter of I.

The directional upper outer density of a set $E \subset H$ at x in the direction $\frac{\pi}{2}$ is defined by

$$\overline{d}^*(E,x) = \lim_{r\to 0} \sup \frac{M^*(E \cap L(x,r))}{r}$$

In particular, if the sets concerned are measurable then M* and d* will be replaced by M and d, respectively.

Let $f : H \to W$, where W is a topological space. The strong essential cluster set $C_s(f,x)$ of f at x is the set of all $w \in W$ such that for every open set U of W containing w, $\overline{d}^*_s(f^{-1}(U),x)>0$.