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A THEOREM ON SEQUENCES OF DIFFERENTIABLE FUNCTIONS

1 Introduction

Some interesting results dealing with convergence of derivatives are known. We can quote e.g. results of D. Preiss and G.Petruska and M.Laczkovich ([4], [3]) stating that each Baire two function is a pointwise limit of derivatives and each Baire one function defined on a nowhere dense compact set is a uniform limit of derivatives. These results, however, don't say anything about convergence of primitives. Except the well known theorem that under uniform convergence of derivatives $(\lim f_n)' = \lim f_n'$ and some of its localizations, the literature contains few other theorems describing the relationship between f' and g, where $f = \lim f_n$, $g = \lim f_n'$. Here we try to fill in this gap for continuous derivatives by showing that the only thing we can say is $f'(x) = g(x)$ almost everywhere on a dense open set. We also show that this assertion holds in the more general case where derivatives of higher orders are considered. As a consequence we get a result related to the aforementioned theorems, namely: for every $p+1$ functions from the first Baire class defined on a nowhere dense closed set there exists a sequence of p-times continuously differentiable functions, such that the sequences of the successive derivatives converge to the corresponding function.

2 Statement of Results

The main result of this paper is the following:

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