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Gabriel I. Istrate*, Department of Computer Science, University of Rochester†, Rochester, NY 14620, U.S.A., e-mail: istrate@cs.rochester.edu

SUMS OF CONTINUOUS AND DARBOUX FUNCTIONS

1 Introduction

The class $\mathcal{C} + \mathcal{D}$ of the real functions that are the sum of a continuous and a Darboux function has received some atention, [2, 5, 6, 7], due to the fact that its exact characterization is not known. The results obtained so far yield comparisons with some classes of functions having generalized Darboux properties. For instance, one knows that $\mathcal{C}+\mathcal{D} \subset \mathcal{U}$, the class of uniform limits of Darboux functions and the inclusion is strict.

Given an interval I and a set $A \subset \mathbf{R}$, denote by $\mathcal{D}^*(I, A)$ the set of all $f : I \longrightarrow \mathbf{R}$ such that range$(f) = A$ and $cl(f^{-1}(y)) = I$ for any $y \in A$ (we will frequently omit I from this notation in the case $I = \mathbf{R}$). In their paper [5] Natkaniec and Kircheim have provided an $f \in \mathcal{D}^*(\mathbf{R}\backslash\mathbf{Q})$ such that $f \notin \mathcal{C}+\mathcal{D}$. The following question arises naturally: *characterize those sets $A \subset \mathbf{R}$ such that $\mathcal{D}^*(I, A) \subset \mathcal{C}+\mathcal{D}$.* Refining the result from [5], we will settle this question.

2 Our result

Clearly any interval (including $\mathbf{R}$ or singleton sets) is a solution of the previous problem. The interesting fact is that there are no other solutions:

Theorem 1 *The only sets $A \subset \mathbf{R}$ for which $\mathcal{D}^*(I, A) \subset \mathcal{C} + \mathcal{D}$ are the intervals.*

PROOF. Suppose there were a set A, other than an interval, having the desired property. Then $\mathcal{D}^*(I, A) \subset \mathcal{U}[I]$. Since the functions from $\mathcal{U}[I]$

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