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SOME EQUIVALENTS OF THE *AP* CONTROLLED CONVERGENCE THEOREM, THEIR GENERALIZATIONS AND A RIESZ-TYPE DEFINITION OF THE *AP*-INTEGRAL

In this paper, the author will propose the definitions of *ap* variational convergence and an *ap* equi-integrable sequence. Their corresponding convergence theorems will be proved to be equivalent to the *AP* Controlled Convergence Theorem. By their equivalency, we prove the condition (3) of the *AP* Controlled Convergence Theorem is actually implied in other conditions. Then we will give some generalizations.

Finally, a Riesz-type definition of the *AP*-integral will be given.

These definitions and theorems are extensions of the oscillation convergence, equi-integrable sequence, Riesz-type definition, and their corresponding convergence theorems with respect to Henstock Integration (see [4], [8]).

1 Prerequisites and Explanation

Our problems are concerned with one dimensional *AP*-integration. The sets and functions involved are assumed to be Lebesgue measurable. The notation $\mathbb{N}$ means all natural numbers, $\mathbb{R}$ denotes all real numbers, $[a, b]$ stands for a bounded real closed interval, and (a, b) is bounded real open interval.

The details of the following definitions and theorems are mainly from [1], [4] Section 22, [5] and [7] Chapter 7.8.

$S = \{S_x : x \in E\}$: We call a measurable set $D_x \subset [a, b]$ an approximate neighbourhood (*ap* neighbourhood) if it has density 1 at x (or has x as a point of density, see [7]) and includes x. Given a measurable set $E \subset [a, b]$, if for every $x \in E$, and *ap* neighbourhood of x, $S_x \subset [a, b]$ is chosen, then we say

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