Real Analysis Exchange
Vol. 20(2), 1994/5, pp. 379–382

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WHERE ANALYSIS, TOPOLOGY AND SET THEORY MEET: WHICH MATHEMATICAL OBJECTS CAN BE INTERESTING FOR TOPOLOGISTS?

The apparent success of topology as a branch of mathematics can be undoubtedly attributed to its wide applicability to the problems originated from the other parts of mathematics. Such applications include a large variety of results from the theory of real functions leading to the creation of functional analysis. All these early applications, however, were limited to the objects (like normed vector spaces) in which the topological structure was naturally existing. Can we apply topological methods for other natural "non-topological" mathematical objects? Can such "non-topological" objects be "made topological?" Here, we "make object topological" by finding a topological structure on a space (or spaces) involved from which the object under consideration can be defined in purely topological terms. For example, a family $\mathcal{G}$ of subsets of a set X can be made topological by finding a topology τ on X such that $\mathcal{G}$ is equal to either of: the topology τ, the family of all τ-closed sets, the family of all τ-Borel sets, the family of all G_δ sets, the ideal of all τ-nowhere dense sets, the σ-ideal of all τ-meager sets, etc. Similarly, a family $\mathcal{F}$ of functions from a set X into a set Y can be made topological by finding the topologies σ and τ on X and Y, respectively, such that the family $\mathcal{F}$ is equal to the family of: all continuous functions from (X,σ) into (Y,τ), all Baire one functions, all Borel functions, etc.

This note sketches the recent study in this direction. It is based on articles [2], [1], and [3] and consists on three corresponding parts.

Which classes of real functions can be topologized? In particular, can we topologize the following classes: Δ – of differentiable functions, $\mathcal{A}$ – of analytic functions, $\mathcal{P}$ – of polynomials, or $\mathcal{L}$ – of linear functions $f(x) = ax+b$?

Theorem 1 *Let $C^\infty \subset \mathcal{F} \subset \mathbb{R}^\mathbb{R}$ be $\circ$-closed. If $\mathcal{F}$ can be topologized then $\mathcal{F}$ is closed under max and min. In particular, classes C^∞ and Δ cannot be topologized.*