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Kurzweil–Henstock Integration and the Strong Lusin Condition

Lee Peng Yee [LPY] in connection with the study of ACG^* functions employed a condition which lies somewhere between absolute continuity and the Lusin condition N, and called it the strong Lusin condition. This condition was also studied by Kurzweil, Jarnik and Schwabik. The aim of this talk is to indicate how this condition can be used in an alternative approach to KH-integration. All functions in the sequel are real valued and measure always means Lebesgue measure.

Definition of SL. *A function F is said to satisfy the Strong Lusin Condition, or briefly SL, on a set S if for every set E of measure zero, and every positive ε there exists a $\gamma : S \to (0, \infty)$ such that for any γ-fine partial division $D = ([u, v], \xi)$ with $\xi \in E \cap S$ we have $(D) \sum |F(u, v)| < \varepsilon$, where $F(u, v)$ denotes $F(v) - F(u)$.*

An application of Vitali's covering theorem makes it possible to show that SL implies N. We denote by N_δ the set of zeros of a function δ. A function $\delta : [a, b] \to [0, \infty)$ will be called a gauge if N_δ is of measure zero.

Definition of the SL-integral. *A function f is said to be SL-integrable on $[a, b]$ if there exists an SL-function F and for every positive ε there is a gauge δ such that $|(D) \sum [f(\xi)(v - u) - F(u, v)]| < \varepsilon$ for every δ-fine partial division D of $[a, b]$. The number $F(a, b)$ is then the SL integral of f and it is denoted by $SL \int_a^b f$.*

Roughly speaking Henstock's lemma is already incorporated in the definition of the SL-integral, an idea already used by Pfeffer in his work on the Gauss-Green Theorem. It can be shown that F from the definition is uniquely determined (up to an additive constant) and that the SL-integral is well defined. It is easy to prove that a KH-integrable function is SL-integrable and it is possible to prove the converse. The KH and SL integrals are equivalent. The concept of the SL-integral allows some simplifications in some proofs of the KH theory. This perhaps could be seen from the theorems that follow. The proofs will appear elsewhere.