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THROWING A DART AT FREILING'S ARGUMENT AGAINST THE CONTINUUM HYPOTHESIS

In his article "Axioms of Symmetry: Throwing Darts at the Real Number Line." Freiling $\begin{bmatrix} 3 \end{bmatrix}$ purports to "give a simple philosophical 'proof' of the negation of Cantor's continuum hypothesis (CH)." The purpose of the present note is to show mathematically why Freiling's argument is not persuasive.

Freiling proceeds as follows in his case against CH:

"Suppose we were to throw a random dart at the real number line (i.e., the interval [0,1]) and ask whether the dart landed on a rational number. The outcome is, of course, predictable. We could say in advance that the dart will (with probability one) land on an irrational number. Furthermore, let us agree that the reason does not depend on any particular property of the set of rational numbers except that it is countable and its members are determined before we make our throw.

"Now suppose we were to throw two random darts and ask whether the second dart was a rational multiple of the first one. The answer would likewise be no, since by the time we throw the second dart there are only countably many

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