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Functions with all singular sets of Hausdorff dimension bigger than one

Introduction. To obtain an integration process which leads to a very general divergence theorem W. F. Pfeffer [P] introduced the c-integral. The domain of this integration is the family of sets with bounded variation (BV sets), [Fe], [G]. During the definition of the $c-$integral first an averaging process, called $v-$integral, is defined on BV sets. Then using an extension method due to Mařik the $v-$integral is extended to the $c-$integral. The extension is necessary because the v-integral is not additive. In fact there exists a BV set $H \subset [0,1]^2$ and a function f defined on $[0,1]^2$ such that f is $v-$integrable on H, and $[0,1]^2 \setminus H$ but not on $[0,1]^2$.

To keep the notation as simple as possible and to avoid technical difficulties instead of BV sets we shall use BVS sets, that is, unions of finitely many squares and points. In this case it is obvious what we mean by the perimeter and the essential boundary of a BVS set.

Since the structure of the BVS sets is quite simple, our example might be useful in other generalizations of the Lebesgue integral.

When dealing with generalized integrals, one has to find out whether Riemann type sums for f can be well approximated by a suitable primitive function. In the definition of the $v-$integral a thin set, that is a set of small Hausdorff dimension, is dropped and one has to check the accuracy of the above approximation modulo thin sets. This motivates