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POROUS SETS AND ADDITIVITY OF LEBESGUE MEASURE

The aim of this paper is to compare some set-theoretic cardinal characteristics of the ideal of σ-porous sets with those similar cardinals associated with the notions of measure and category which have been extensively studied by A. W. Miller, J. Ihoda, S. Shelah, and others. We will prove that every set of reals of cardinality less than the additivity of the ideal of Lebesgue measure zero sets is σ-porous, the real line can be covered by a family of closed porous sets of cardinality of any cofinal family of the ideal of Lebesgue measure zero sets and it is consistent that the minimal cardinality of a set which is not σ-porous is greater than the minimal cardinality of an unbounded family of functions from ${}^{\omega}\omega$. In fact, we will prove all these for the ideal of σ-strongly symmetrically porous sets.

If A is a subset of the real line $\mathbb{R}$, $I = (a,b)$ is an open interval then we denote by $\lambda(A,I)$ the length of the largest open subinterval of I which does not intersect A and $\lambda^*(A,I)$ is the largest $\delta \geq 0$ such that $(a,a+\delta) \cup (b-\delta,b)$ is disjoint with A. The porosity and the symmetric porosity of A at $c \in \mathbb{R}$ is the number

$p(A,c) = \limsup_{\varepsilon\to 0+} \lambda(A,(c-\varepsilon,c+\varepsilon))/\varepsilon$ and

$s(A,c) = \limsup_{\varepsilon\to 0+} \lambda^*(A,(c-\varepsilon,c+\varepsilon))/\varepsilon$, respectively.

We say that A is porous (resp. strongly porous, resp. strongly symmetrically porous) if $p(A,a) > 0$ (resp. $p(A,a) = 1$,