Real Analysis Exchange Vol 15 (1989-90)

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ON THE EQUIVALENCE OF HENSTOCK-KURZWEIL AND RESTRICTED DENJOY INTEGRALS IN R^n

1. Introduction

Several descriptive definitions of the restricted Denjoy integral in R^n were given in the 1930's and 1940's [3, 8, 9], which are in terms of the generalized absolute continuity of the primitive function. However, for the next forty years, none of these has been proved to be equivalent to the Henstock-Kurzweil integral or Perron integral in R^n, except for the case when $n = 1$ [4, 6, 12]. For a recent attempt, see [10, p.83]. In this note, we shall settle the above problem, and, as a consequence, the problem posed by Pfeffer in [16, Problem 6.6]. Pfeffer's problem is: Give a Denjoy type descriptive definition of HF-integrals defined in section 4.

The Henstock-Kurzweil integral is of Riemann-type, which is defined by simply replacing the fixed norm δ in the Riemann integral by a positive function $\delta(x)$. This basic idea of replacing the constant δ of the classical definition by a positive function has been explored in many fields, for example, in integration theory [1, 6, 11, 15], in variation theory [5, 18] and in covering theory [2, 18]. The generalized absolute continuity of this type, denoted by ACG^*, is defined by Henstock in [6, p.58], which was drawn to the author's attention by P. Y. Lee. This Henstock version of ACG^* is equivalent to the classical ACG_* [17, Chapter VII] in the one-dimensional space R. With this Henstock version, the equivalence of the Henstock-Kurzweil and the restricted Denjoy integrals in R^n can be proved easily. Furthermore, many proofs in R can be shortened. The Henstock version is the genuine one whereas the classical version in R is the resultant of the Henstock version and the property of