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Differentiability and Density Continuity

1 Introduction

The density topology [10,11] on $\mathbf{R}$ consists of all measurable subsets A of $\mathbf{R}$ such that, for every $x \in A$, x is a density point of A. It is a completely regular refinement of the natural topology. A function $f\colon \mathbf{R} \to \mathbf{R}$ is *density continuous* if and only if it is continuous as a selfmap of $\mathbf{R}$ equipped with the density topology. The class of density continuous functions was investigated by Ostaszewski [7,,8]. Bijections of the real line whose inverses are density continuous were studied by Bruckner [1] and Niewiarowski [6]. Ostaszewski [9] considered the class as a semigroup with composition as the operation. Ciesielski and Larson [2] showed that real-analytic functions are density continuous, and that the class of density continuous functions is not a linear space. Furthermore, there exist C^∞ functions which are not density continuous. Ciesielski, Larson, and Ostaszewski [4] proved that a typical continuous function is nowhere density continuous, and the class of sets of points of discontinuity of density continuous functions is that of nowhere dense F_σ subsets of $\mathbf{R}$.

Throughout this paper we are concerned with the relationship between density continuity and differentiability. In the process, we discuss the fact that any closed set can be made into the zero set of a C^∞ density continuous function, and we show that there is a nowhere approximately differentiable

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