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THE CHI FUNCTIONS IN GENERALIZED SUMMABILITY

In [2], Baric defined, for conservative matrices, a generalized summability analogue to the chi functional of scalar summability. This function, together with analogues to the functionals χ_n, may be defined for any conservative transformation T. These functions have multiplicative properties similar to those established for the chi functionals in [12, section 3]. We use these properties to give a necessary and sufficient condition for an invertible conservative matrix to have a matrix inverse. We also use the chi functions to make algebraic statements about certain algebras of conservative matrices and to show that some of them are Banach algebras. We close with remarks on certain algebras which contain conull matrices.

Our notation and terminology are standard. Let E be a Banach space. The spaces $m(E)$, $c(E)$, and $c_0(E)$ consist, respectively, of bounded sequences in E, convergent sequences in E, and null sequences in E. If $E = \mathbf{C}$ is the complex numbers, we write m, c, and c_0. Each of these spaces is a Banach space under the norm $||x|| = sup||x_k||$, $x = \{x_k\}$ a sequence in E. The coordinate functions C_n defined by $C_n(x) = x_n$ are continuous on these spaces. Baric [2] calls these spaces FK spaces since they are Frechet spaces with continuous coordinates. The space $\ell^1(E)$ consists of those sequences x in $c(E)$ for which $\sum ||x_k||$ is finite.

Let F be a second Banach space. A continuous linear transformation T from $c(E)$ to $c(F)$ is called *conservative* . If T can be represented by an infinite matrix $A = (A_{nk})$, n and k positive integers, where each A_{nk} is a continuous linear transformation from E to F, then T is called a conservative *matrix*. Conservative matrices are characterized in [1, Proposition 1.2](the characterization is due originally to Robinson [9] and Melvin-Melvin [7]). The set of conservative matrix transformations from $c(E)$ to $c(F)$ is denoted