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RADIAL CLUSTER SET AND INTERPOLATION

Let D be the open unit disk in the complex plane C and f be an analytic function from D into C. Denote by S the unit circle in the complex plane. For each σ in S, denote the radius from 0 to σ by $R(\sigma)$. That is, $R(\sigma) = \{t\sigma : 0 \leq t < 1\}$. We are interested in the behavior of f restricted to $R(\sigma)$. In general, the limit of $f(t\sigma)$ as t tends to 1 does not exist. But, as a function into the extended complex plane C^* (that is, the Riemann sphere), the radial cluster set $C(f,\sigma)$ does exist, where $C(f,\sigma)$ is the subset of C^* given by

$$C(f,\sigma) = \cap_{1>t\geq 0} \mathrm{Cl}(\{f(\tau\sigma) : t \leq \tau < 1\}),$$

where Cl(E) is the closure in C^* of the subset E of C^*. The continuity of f assures that $C(f,\sigma)$ is a nonempty subcontinuum of C^*. The collection of all nonempty subcontinua of C^* will be denoted by $C(C^*)$. The function f^* defined on S into $C(C^*)$ given by $f^*(\sigma) = C(f,\sigma)$ is called the <u>radial cluster set function of f</u>.

Corresponding to the metric on C^*, there is a metric on the collection $C(C^*)$ called the Hausdorff metric. By 1938, it was known that $C(C^*)$ with this metric is a Peano continuum (a connected, locally connected, compact metric space). In 1974, Curtis and Schori ([1] and [2]) proved a long standing conjecture of Wojdyslawski [3] on C(X), where X is a Peano continuum. As a result of this theorem of Curtis and Schori, we now know that $C(C^*)$ is homeomorphic to the Hilbert cube.