

MEASURABLE DARBOUX FUNCTIONS

Consider the following "topologically defined" classes of functions $f : [0,1] \rightarrow \mathbb{R}$,

- EXT: f is extendable to a connectivity function $g : [0,1] \times [0,1] \rightarrow \mathbb{R}$,
- AC: f is almost continuous, i.e. every open set containing the graph of f contains the graph of a continuous function with domain $[0,1]$,
- Conn: f is a connectivity function, i.e. $f|_C$ is connected for every connected subset C of $[0,1]$,
- D: f is Darboux, i.e. $f(C)$ is connected for every connected subset C of $[0,1]$,
- PC: f is peripherally continuous if for each x and each pair of open sets U and V containing x and $f(x)$, respectively, there is an open subset W of U containing x such that $f(\text{bd}(W))$ is a subset of V ,
- PR: f has a perfect road at each point if for each x , there is a perfect set having x as a bilateral limit point such that $f|_P$ is continuous at x ,
- Z_c : all sets of the form $\{f < t\}$ and $\{f > t\}$ are either empty or bilaterally c -dense in themselves,
- Z_ω : all sets of the form $\{f < t\}$ and $\{f > t\}$ are either empty or bilaterally dense in themselves.

We discuss the relationships that are known to exist between these "topologically defined" classes of functions within certain "measurability classes":

- B_1 : Baire class 1,
- R_1 : pointwise limits of right-continuous functions,
- J_1 : pointwise limits of functions with only "jump-discontinuities",
- G_δ : functions with G_δ graphs,
- B: Borel functions,
- U: universally measurable functions,