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Lifting: the connection between functional representations of vector lattices

(summary)

The Ogasawara-Maeda [**LZ**] theorem states that every Archimedean vector lattice (= Riesz space) is representable as a subspace of the densely finite, continuous extended real-valued functions on some extremally disconnected compact Hausdorff topological space. I. Fleischer [**F2**] has recently given a new proof of this by passing through a representation of Carathéodory's place ("pointless") functions. These latter can also be realized as (co-meagrely finite) measurable functions modulo those which vanish outside of members of a σ-ideal via the Loomis-Sikorski theorem, from which Fleischer obtains another representation of the vector lattice. The use of a certain Stone space in both cases leads to the feeling that the representations must be "the same". We verify this, and give the connection with lifting theory of measure spaces.

Let $\mathcal{B}$ denote the σ-field of *Baire-property* (i.e., having a meagre symmetric difference with some open set) sets in an extremally diconnected (open sets have open closures) Baire space, or more generally, the σ-field of sets differing by a meagre from a cozero set in a basically disconnected [**GJ**] (cozero sets have open closures) in a basically Baire space (no nonvoid meagre cozero sets). Denote by

$\mathcal{L}^0$	the space of $\mathcal{B}$-measurable real functions
$\mathcal{N}_F$	its ideal of co-meagrely null functions
$\mathcal{L}^\infty$	its subspace of co-meagrely bounded functions
$\mathcal{C}$	the continuous bounded functions
$\mathcal{C}_\bullet$	the continuous densely finite extended real-valued functions.

THEOREM.

(1) *There is a lifting c of $\mathcal{B}$ modulo the meagre sets, with image the Boolean algebra of clopen sets.*

(2) *Integration with respect to $\lambda(A) = \mathbf{1}_{c(A)}$ is a strong lifting of $\mathcal{L}^\infty$ modulo $\mathcal{N}_F$, rendering $\mathcal{C}$ and $L^\infty = \mathcal{L}^\infty/\mathcal{N}_F$ isometrically isomorphic, and*

(3) *is a strong essential lifting of $\mathcal{L}^0(\mathcal{B})$ modulo its co-meagrely null functions, rendering $\mathcal{C}_\bullet$ and $L^0 = \mathcal{L}^0/\mathcal{N}_F$ isomorphic.*

Thus, c is an idempotent Boolean homomorphism on $\mathcal{B}$ onto its subalgebra of clopens with kernel the meagres; integration is an idempotent vector lattice (and ring) homomorphism on $\mathcal{L}^\infty$ selecting a (the) continuous bounded representative out of each class modulo the null functions,

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