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I-density Continuous Functions

Here, and in what follows I will stand for the ideal of first category subsets of R.

Definition of an I-density point of a set $A \subseteq R$.

Motivation: 0 is a density point of A iff

$$\lim_{n\to\infty} \frac{m\left[A\cap\left(-\frac{1}{n},\frac{1}{n}\right)\right]}{\frac{2}{n}} = 1 \quad \text{iff} \quad \lim_{n\to\infty} m\left[n\cdot A\cap(-1,1)\right] = 2 \quad \text{iff}$$

$$\chi_{n\cdot A\cap(-1,1)} \underset{n\to\infty}{\to} \chi_{(-1,1)} \quad \text{in measure iff}$$

$$\forall(n_m)\exists(n_{m_k}) \lim_{k\to\infty} \chi_{n_{m_k}\cdot A\cap(-1,1)} = \chi_{(-1,1)} \quad I_m\text{-a.e.}$$

Definition (Wilczyński)

(1) 0 is an I-density point of A iff

$$\forall(n_m)\exists(n_{m_k}) \lim_{k\to\infty} \chi_{n_{m_k}\cdot A\cap(-1,1)} = \chi_{(-1,1)} \quad I\text{-a.e.}$$

(2) x is an I-density point of A iff
0 is an I-density point of $(-x)+A$

Let τ_I be a family of all Borel sets $A\subset R$ such that every point $x\in A$ is an I-density point of A.