RESEARCH ARTICLES Real Analysis Exchange Vol. 13 (1987-88)
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MAXIMAL ADDITIVE FAMILIES FOR SOME CLASSES OF DARBOUX FUNCTIONS

Preliminaries. Let $C^+(f,x)$ and $C^-(f,x)$ denote the set of all right-side and left-side limit numbers of the function f at the point x. For any subset M of the plane $\mathbb{R}^2$, $cl(M)$ denotes the closure of M and $card(M)$ denotes the cardinality of M. No distinction is made between a function and its graph.

In [2] Bruckner and Ceder described what it means for a real function to be Darboux at a point. We say that a function f is Darboux from the right-side [left-side] at a point x (we write $x \in D_+(f)$, $x \in D_-(f)$ respectively) if and only if

1° $f(x) \in C^+(f,x)$ $[f(x) \in C^-(f,x)]$; and

2° whenever $a,b \in C^+(f,x)$ $[a,b \in C^-(f,x)]$ and y is any point between a and b, then for every $\varepsilon > 0$ exists a point $\xi \in (x,x+\varepsilon)$ $[\xi \in (x-\varepsilon,x)]$ such that $f(\xi) = y$.

In [3] Császár showed that a function is Darboux if and only if it is Darboux at each point.

By D^c we denote the class of Darboux functions whose upper and lower boundary functions are continuous. By D^* we denote the class of functions which take on every real value in every interval and by D^{**} we denote the class of functions which take on every real value c-times in every interval, where c denotes the cardinality of the continuum.

It is clear that $D^{**} \subset D^* \subset D^c$.

For a given family F of real functions let $M(F)$ denote the class of all functions g such that $f \in F$ implies $f + g \in F$. This class is called the maximal additive family for F. It is known [1] that the family of continuous functions C is the maximal additive family for the class of Darboux Baire 1 functions DB_1 and the family of constant functions K is the maximal additive family for the class of Darboux functions D.