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Some Answers to a Question of P. Bullen

A question posed by Peter Bullen [1] is whether it is
possible to restrict the guage function used in generalized
Riemann type integrals. Here we investigate the guage function
needed for the Perron integral (equivalent to the Riemann-Complete
and narrow sense Denjoy integral), the Lebesgue-Stieljes integral
and the Lebesgue integral for bounded measurable functions. The
Henstock or Riemann-Complete (R-C) integral integrates a
function f assumed to be finite valued. The R-C integral of
f on [a,b] is L provided that for each $\varepsilon > 0$ there is a
positive function δ such that $\left| \sum f(z_i) \Delta x_i - L \right| < \varepsilon$ whenever
$a = x_0 < x_1 < \dots < x_n = b$ is a partition of [a,b],
$z_i \in [x_{i-1}, x_i]$ and $\Delta x_i = x_i - x_{i-1} < \delta(z_i)$. The function δ
is called the guage function.

Henstock [2, p.127] showed how to determine δ for the
Perron integral. Utilizing the majorant and minorant, he found
δ and showed that the Perron integral is contained in the R-C
integral. However, the character of δ has not been determined.