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A FUNCTION IN THE DIRICHLET SPACE B SUCH THAT ITS FOURIER SERIES DIVERGES ALMOST EVERYWHERE

An analytic function F on the disc belongs to the Dirichlet space B if $\|F\|_B = \int_0^1 \int_0^{2\pi} |F'(re^{i\theta})| d\theta dr < \infty$. Notice that $B \subsetneq H^1 \subsetneq L^1$, where H^1 is the Hardy space of all analytic functions F on the disc so that $\|F\|_{H^1} = \sup_{0<r<1} \int_0^{2\pi} |F(re^{i\theta})| d\theta < \infty$, and L^1 is the Lebesgue space of all integrable functions on $[0,2\pi]$. The inclusion $H^1 \subsetneq L^1$ is taken in the sense of boundary values, that is, $F \in H^1$ implies $\lim_{r\to 0} \operatorname{Re} F(re^{i\theta}) \in L^1$.

Kolmogorov [Une serie de Fourier-Lebesgue divergente presque partout, Fund. Math. 4 (1923), 324-8] showed that there exists an f in L^1 such that its Fourier series diverges almost everywhere. Also, it was shown by Sunouchi [A Fourier series which belongs to the class H^1 diverges almost everywhere, Kodai Math. Sem. Rep. 1 (1953) 27-8], who modified an example of Hardy and Rogosinski [Fourier Series, Cambridge Tracts #38, 2nd Edition, 1949], that there is a function in H^1 with a divergent Fourier series. Therefore, a natural question to ask is: Do the functions in B have convergent Fourier series.

This question was asked by Professor Guido Weiss to the first named author.

In this lecture we answer this question negatively.