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THE RESTRICTIONS OF A CONNECTIVITY FUNCTION ARE NICE BUT NOT THAT NICE

Let X and Y be topological spaces. A function $f:X \to Y$ is said to be a connectivity function provided that if A is a connected subset of X, then the graph of f restricted to A is a connected subset of $X \times Y$. A function $f:X \to Y$ is said to have property (s) or to be (s)- measurable provided that for each perfect set $P \subset X$ there exists a perfect set $Q \subset P$ such that the restriction $f|Q$ is continuous. Marczewski defined property (s) for sets in [8] and showed that (s)- measurable functions and the class of functions (functions with property (s)) studied by Sierpinski in [10] were the same. For further study the reader is referred to [1] and [2]. A real-valued function f defined on an interval is said to have a perfect road at the point x provided that there exists a perfect set P such that x is a bilateral point of accumulation of P and such that $f|P$ is continuous at x.

Let $I = [0,1]$. From [4] it follows that if $g:I^2 \to I$ is a connectivity function, then $f = g|(I \times \{x\})$ has the following property: if $[a,b] \subset I$, then there exists a Cantor set $C \subset (a,b)$ such that $f|C$ is continuous where I is assumed to be embedded in I^2 as $I \times \{x\}$ for any $x \in I$. From [6] it follows that if $g:I^2 \to I$ is a connectivity function, then $f = g|(I \times \{x\})$ has a perfect road at each point for any $x \in I$. However, there exist connectivity functions $I \to I$ that