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L-POINTS OF TYPICAL FUNCTIONS IN THE ZAHORSKI CLASSES

Functions considered in this paper will belong to the space $\mathcal{B}^1$, the space of Baire class one functions on the interval $[0,1]$ equipped with the metric of uniform convergence. Ever since Lebesgue [6], it has been known that any function in the space of bounded Baire class one functions on $[0,1]$, $b\mathcal{B}^1$, is the derivative of its indefinite integral except at a set of points which is both of measure zero and of first category. As in [4], for $f \in \mathcal{B}^1$, we call x an L-point of f if $\lim_{h\to 0} \frac{1}{h}\int_0^h f(x+t)dt = f(x)$, and we let

$$N(f) = \{x \in [0,1]: \lim_{h\to 0} \frac{1}{h}\int_0^h f(x+t)dt \text{ does not exist}\}.$$

Although the set of L-points for any function in $b\mathcal{B}^1$ is large in terms of measure and category, it was shown in [5] that for the typical (in the sense of category) function $f \in b\mathcal{B}^1$ the set $N(f)$ fails to be σ-porous. More specifically, it was shown in that paper that if $\mathcal{N} = \{f \in \mathcal{B}^1: N(f)$ is σ-porous$\}$, and if $\mathcal{F}$ is any of the spaces $\mathcal{B}^1$, $b\mathcal{B}^1$, $\mathcal{B}^1\mathcal{D}$ (the Baire one Darboux functions), or $b\mathcal{B}^1\mathcal{D}$, then $\mathcal{N} \cap \mathcal{F}$ is closed and nowhere dense in $\mathcal{F}$.

Thus, using Zahorski's [11] notation, we have the situation that the typical function in the space $b\mathcal{M}_0 = b\mathcal{M}_1 = b\mathcal{B}^1\mathcal{D}$ has a non-σ-porous set of