

Sandra Meinershagen, Department of Mathematics

University of Missouri at Kansas City, Kansas City, MO 64110

$D^\#$ Derivation Basis and the
Lebesgue-Stieltjes Integral

This paper answers a question posed by Thomson [7, p.377] on the relation between the $D^\#$ derivation basis and the Lebesgue-Stieltjes integral. The question is stated as follows: "The basis $D^\#$ can be used to characterize the Lebesgue integral. The corresponding Stieltjes integral $D^\# \int f(x)dg(x)$ seems not to have been investigated, apart from several remarks in McShane." The remark in McShane, while vague, asserts that if the $D^\#$ Stieltjes integral exists for continuous functions f with respect to a function g , then g must be of bounded variation [2, p.40]. It will be shown that the $D^\# \int f(x)dg(x)$ is the Lebesgue-Stieltjes integral for functions g of bounded variation.

The following definitions and notation will be needed. They are taken from Thomson [7, pp.87, 92, 101, 108, 116, 117, 125, 137, 157, 165].

Definition 1.1. The sharp derivation basis. For a positive function δ on R ,

$\beta_\delta^\# = \{(I,x): I \text{ is an interval in } R, x \in R, \text{ and}$

$I \subset (x-\delta(x), x+\delta(x))\}$ and