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ON A RESULT OF S. KUREPA

Introduction

In an article published in 1956, S. Kurepa [2] proved the following theorem.

Theorem. *There exist Lebesgue measurable sets* $A, B \subset R^n$ *such that the set* $A + B = \{a + b: a \in A, b \in B\}$ *is nonmeasurable.*

Here $a + b$ is the ordinary coordinate wise sum of a and b, i.e. if $a = (a_1, a_2, \ldots, a_n)$ and $b = b_1, b_2, \ldots, b_n)$ then $a+b = (a_1+b_1, a_2+b_2, \ldots, a_n+b_n)$.

The proof of this theorem can be found in M. Kuczma's new book "An Introduction to the Theory of Functional Equations and Inequalities" ([1], pg. 256). Kuczma introduces Kurepa's theorem, saying it "shows a certain irregularity of the operation +". The purpose of this paper is to extend Kurepa's result by showing that a wide class of operations on R^n (i.e. functions on $R^n \times R^n$ into R^n) actually share the irregularity of the operation + noted above.

Before presenting our results we mention that the sets A and B in Kurepa's paper (and in Kuczma's book) are constructed using a measurable Hamel basis and that this construction can not be extended to show a similar result for operations different from +. Furthermore, Kurepa's sets A and B both turn out to be sets of Lebesgue measure zero.

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