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Borel measurability of extreme path derivatives

The derivative F' of any differentiable function F is a function of Baire class one, since F' is the pointwise limit of a sequence of continuous functions $\{n(F(n+\frac{1}{n})-F(x))\}_{n=1}^{\infty}$. A problem that dates back to the beginning of this century is that of finding the Baire classification of various types of extreme derivatives. Sierpinski [11] showed that the Dini derivatives of a function of Baire class α are in Baire class $\alpha + 3$. Banach [2] proved that the Dini derivatives of the bounded functions of Baire class α are in Baire class $\alpha + 2$. We have also by Kempisty [5] and Hajek [4] the successive results that the extreme bilateral derivatives (for arbitrary functions) are in Baire class 3 and in Baire class two. Misik [7] was able to generalize Banach's result for arbitrary functions of Baire class α. He showed that the upper (lower) Dini derivatives of a Borel function of Baire class α are upper (lower) semi-Borel functions of Baire class $\alpha + 1$. He also [8], [9] proved that for any ordinal number α the upper (lower) unilateral approximate derivatives of Borel functions of the class α are lower (upper) semi-Borel functions of the class $\alpha + 2$.

Bruckner, O'Malley and Thomson [3] introduced the concept of path derivative as a unifying approach to the study of a number of generalized derivatives. They showed that for a system of paths