C.T. Ng, Department of Pure Mathematics, University of Waterloo, Waterloo, Ontario N2L 3G1

ON CONVEXITY

The following theorem is a summary of the talk given at the Tenth Summer Real Analysis Symposium.

Theorem. *Let* $f: \mathbb{R} \to \mathbb{R}$. *Then the following statements are equivalent:*

a) f *generates a Schur-convex sum* $\sum_1^n f(x_i)$ *on* $\mathbb{R}^n$;

b) f *is Wright-convex, i.e.* $f(x+\delta) - f(x) \leq f(y+\delta) - f(y)$ *for all* $x < y$, $\delta > 0$;

c) f *has the representation* $f = C + A$, *where* $C: \mathbb{R} \to \mathbb{R}$ *is convex and* $A: \mathbb{R} \to \mathbb{R}$ *is additive* *(i.e.* $A(x+y) = A(x) + A(y)$);

d) f *is midconvex, and is locally bounded from above by a midconcave function at some point.*

The equivalence between a), c) and d) can be extended to functions defined on open convex subsets of $\mathbb{R}^m$, where b) requires extra interpretation. It is well-known from the works of Schur, HLP, that convex functions generate Schur-convex sums; and so the equivalence between a) and c) strengthened such ties. The equivalence between c) and d) solved a problem posed by Nikodem. Wright-convexity is recorded in the book of Roberts and Varberg.

References

1. G.H. Hardy, J.E. Littlewood, and G. Pólya, *Inequalities,* Cambridge University Press, London and New York, 1934, 1952.

2. A.W. Marshall and I. Olkin, *Inequalities: Theory of Majorization and Its Applications*, Academic Press, New York, 1979.