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DERIVATION BASES AND THE HAUSDORFF MEASURE

The purpose of this talk is to answer a question posed by Thomson [5, p.164] on the relation, if any, between the Hausdorff measure and the D derivation basis. It is shown that the Hausdorff measure equals the measure generated by the D derivation basis (1) when the derivative of h at 0 exists and is finite, (2) when the set is countable or (3) when the sum $\Sigma h(|I_n|)$ over the contiguous intervals of a given closed set converges. However, it is shown that the symmetric derivation basis is finite on more sets of finite Hausdorff measure then the measure from the D derivation basis.

When the lower right derivate of h at 0 is finite, the Hausdorff measure is a multiple of the Lebesgue measure. $\underline{D}^+h(0)$ is the multiple. When the upper right derivate of h at 0 is finite, $(h\circ m)_s(E)$, $(h\circ m)_D(E)$ and $(h\circ m)_{D^{\#}}(E)$ are a multiple of the Lebesgue measure and that multiple is $\overline{D}^+h(0)$.

The remaining question as to the relation between the measures is when h has an infinite derivate at 0. It is answered as follows: For the $D^{\#}$ derivation basis, the answer is trivial. If E is any non-empty set in [a,b] then $(h\circ m)_{D^{\#}}(E) = \infty$. As was mentioned above, when the