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CONVERGENCE THEOREMS IN INTEGRATION THEORY

We shall define the Henstock integral [5, 6, 14], describe three convergence theorems [9, 11] and sketch one of the proofs.

A function f is said to be Henstock integrable to A on [a, b] if for every $\varepsilon > 0$ there is a function $\delta(\xi) > 0$ such that for any division (called δ-fine) given by

$$a = x_0 < x_1 < \ldots < x_n = b \text{ and } \xi_1, \xi_2, \ldots, \xi_n$$

satisfying $\xi_i - \delta(\xi_i) < x_{i-1} \leq \xi_i \leq x_i < \xi_i + \delta(\xi_i)$ for $i = 1, 2, \ldots, n$, we have

$$\left| \sum_{i=1}^{n} f(\xi_i)(x_i - x_{i-1}) - A \right| < \varepsilon.$$

It is well-known that the Henstock integral is equivalent to the Denjoy integral [5, 7, 17] and to the Perron integral [5, 15, 17]. The Henstock integral is also known as the Kurzweil integral [8].

Next, we define major and minor functions and functions which are ACG_*. A function H is said to be a major function of a function f in [a, b] if

$$-\infty \neq \underline{D}H(x) \geq f(x) \quad \text{for every } x$$

where $\underline{D}$ denotes the lower derivative. A function G is said to be a minor function of f in [a, b] if $-G$ is a major function of $-f$ in [a, b].

A function F is said to be $AC_*(X)$ if for every $\varepsilon > 0$ there is $\eta > 0$ such that for every finite and infinite sequence of non-overlapping intervals