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INTEGRATION IN FUNCTION SPACES

R. Henstock's general theory of integration is based on division spaces rather than measure theory (1,2). Division spaces arise as follows. Given a space T and a family of subsets or "intervals" I of T, a partition of T is a finite collection of disjoint intervals I whose union is T. Henstock defines collections S of interval-point pairs (I,x), $x \in T$. A division $\mathcal{E}$ of T from S is a finite subcollection of (I,x) from S such that the intervals I form a partition of T. The conditions satisfied by the collections S include the following.

(i) There exists S containing a division of T. (For such S we say that S divides T.)

(ii) If S_1 and S_2 both divide T then there exists S_3, dividing T, in the intersection of S_1 and S_2.

If f is a real or complex valued function of points x in T and m is, similarly, a function of the intervals I of T, then the integral over T of f with respect to m, which we denote by $\int_T f(x)m(I)$ or $\int_T fdm$, is z where z satisfies the following condition.

Given $\varepsilon > 0$ there exists S dividing T so that, for any division $\mathcal{E}$ of T from S, .