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Recent Developments in Fourier Analysis and Generalized Bounded Variation

Bounded variation and its generalizations are associated with many aspects of Fourier series. Here we discuss three recent results dealing with these aspects. The first involves a new notion of summability which arises naturally from the consideration of Fourier series of functions of  $\Lambda$ -bounded variation. The second is concerned with functions whose Fourier series have small gaps and the significance of the assumption that such a function is of generalized bounded variation on a subinterval. The third result is an improvement of the Bohr-Pál theorem: we have shown that if  $f$  is continuous and  $2\pi$ -periodic, then there is a homeomorphism of  $[0, 2\pi]$  onto itself such that  $f \circ g$  is the conjugate of a function of bounded variation.

1. SUMMABILITY OF FOURIER SERIES.

Let  $\Lambda = \{\lambda_k\}$  be a nondecreasing sequence of real numbers with  $\lambda_1 = 1$  and  $\sum_1^\infty 1/k\lambda_k < \infty$ . For a given positive integer  $n$ , let  $I_{k,n} = ((k-1)\pi/n, k\pi/n]$ ,  $k = 1, 2, \dots, n$ . Set

$$H_n(t) = \sum_1^n (k/\lambda_k) \chi_{I_{k,n}}(t) D_n(t)$$

where  $\chi_E$  denotes the characteristic function of a set  $E$  and  $D_n(t)$  is the