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Gabriela Ene and Vasile Ene, Institute of Mathematics, str. Academiei 14, 70109, Bucharest, Romania.

NONABSOLUTELY CONVERGENT INTEGRALS

Interesting generalizations of the descriptive definition for nonabsolutely convergent integrals were given by H.W. Ellis [3], J. Foran [6], and C.M. Lee [11]. The most remarkable one is that of Foran, which is a classical generalization of the Denjoy integral in the wide sense, i.e., Foran's class of primitives is a class of continuous functions which contains strictly the ACG functions. The classes of primitives for the integrals of Ellis and the integrals of Lee are not classes of continuous functions and restricted to the class of continuous functions one obtains at most the class of ACG functions.

In this paper we give various extensions for each of these integrals. The classes of primitives for our generalizations are not classes of continuous functions. However, if one restricts these primitives to the continuous functions, some of these classes contain strictly the primitives in the Foran sense.

The uniqueness of the integration for the Foran integral follows by a corollary of Theorem 7.7 of [12] (p. 285).

To assure the uniqueness of our integrations we give some monotonicity theorems among which Theorem 3 is the most important. Theorem 3 generalizes Theorem 7.7 of [12] (p. 285) and its corollaries are both intrinsically interesting and useful.

For convenience if P is a well-defined property for functions defined on a certain domain, we will also use P to denote the class of all functions having the property P. The conditions (N), T_2, VB_*, VBG_*, VB, VBG, AC_*, ACG_*, AC, ACG are defined in [12]. In [6] Foran introduced conditions A(N) and B(N) and in [5] V. Ene has introduced condition E(N). If in the definition of A(N) the intervals I_k are allowed to overlap, a more restrictive condition results which we call condition $A^\bullet(N)$.

We denote by $\mathcal{F}$ (respectively $\mathcal{F}^\bullet$, $\mathcal{B}$, $\mathcal{E}$) the class of all continuous functions F defined on a closed interval I for which there exist a sequence $\{E_n\}$ of sets and a sequence $\{N_n\}$ of natural numbers such that $I = \cup E_n$ and F is $A(N_n)$ (respectively $A^\bullet(N_n)$, $B(N_n)$, $E(N_n)$) on