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NONABSOLUTE INTEGRATION IN THE PLANE

In this paper certain results from the dissertation [19] will be presented.

In his topical surveys [29] and [30], Brian Thomson introduced a unified approach to nonabsolute integration on the real line, based on the theory of integral due (in the general setting) to Ralph Henstock ([6], [7], [8]), and (in certain specific settings) to Jaroslav Kurzweil ([13], [14]), and E. J. McShane ([18]).

Following this direction, we consider Henstock integrals in the plane. This requires the notion of a derivation base.

1.1. Definition. Let X be a nonempty set and Ψ a nonvoid class of its subsets. A nonempty class

$$\Delta \subset \mathfrak{P}(X \times \Psi) \tag{1}$$

will be termed a *derivation base* on X .

We will usually take X to be \mathbb{R}^2 and Ψ — nondegenerate closed intervals, regular intervals, triangles, etc. In [29] and [30] X is taken to be the real line, and Ψ is the class of all closed nondegenerate intervals.

A more general setting is possible. In [1] an integration theory of Henstock type in a locally compact Hausdorff space is presented. A space A equipped with a class $\{I\}$, as in [4] and [31], is also a possibility. Also, [32] presents nonabsolute integration in topological spaces.

A base Δ is called *trivial* if $\emptyset \in \Delta$. Unless stated otherwise, all bases considered are nontrivial.

Elements of a base Δ will be denoted by small Greek letters $(\alpha, \beta, \gamma, \dots)$.

1.2. We will assume that Ψ has the following property: given $I_0, I_1, \dots, I_n \in \Psi$, and $I_1, \dots, I_n \subset I_0$,

$$I_0 \setminus (I_1 \cup I_2 \cup \dots \cup I_n) = J_1 \cup J_2 \cup \dots \cup J_m \tag{2}$$