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On Typical Bounded Functions in the Zahorski Classes II

In [5], this author investigated properties of typical functions in terms of derived numbers. Here, we continue and extend our investigation to include typical properties stated in terms of intersections of graphs with straight lines.

All functions will be real valued with domain I=[0,1]. Zahorski, in [6], defined a nested hierarchy of classes of functions, $M_1 \supset \ldots \supset M_5$, and showed that M_1 is the class of Darboux Baire 1 functions ($\mathcal{DB}_1$) and M_5 is the class of approximately continuous functions. For i=1,...,5, the class of bounded M_i functions (bM_i) is a complete metric space under the sup norm, so by a typical function we mean one belonging to a residual subset of bM_i.

The associated sets of a function, f, are sets of the form $\{x|\, f(x)>a\}$ and $\{x|\, f(x)<a\}$ for a real. We let $D_L f(x)$ and $D_R f(x)$ denote the set of derived numbers of f at x on the left and right respectively. The Lebesgue measure of a set A will be $\lambda(A)$, and R (resp. R^*) will mean the set of real (resp. extended real) numbers. By $C^-(f,x)$ and $C^+(f,x)$ we mean the left and right cluster sets of f at x. Let t_f (resp. b_f) be the supremum (resp. infimum) of the set $C^-(f,x) \cup C^+(f,x)$.

In [5], we showed that the typical bM_i function has every extended real number as a derived number at every point. That is, $D_L f(x) \cup D_R f(x) = R^*$ for every x in I, the obvious modifications made