

Stanislav Vaněček, Department of Math. Analysis, Charles University,
Sokolovská 83, 186 00 Praha, Czechoslovakia

Solution of a Problem Concerning Functions of Harmonic Bounded Variation

It is known [3] that the space of all regulated functions for which the Fourier series converges for every change of variable (GW) contains the space of functions of harmonic bounded variation (HBV) and the space of all functions for which the Fourier series converges uniformly for every change of variable (UGW) contains the space of continuous functions of harmonic bounded variation (HBV_c). In [3, p. 17] and [4] the question is raised whether $GW = HBV$ and $UGW = HBV_c$. In [5] it is pointed out that [1] contains the result $HBV_c \subseteq UGW \subsetneq GW_c$, implying $HBV \subsetneq GW$. The purpose of the present note is to prove that $HBV_c \subsetneq UGW$. I would like to express my thanks to RN Dr. L. Zajíček for acquainting me with the problem.

The function $f : \mathbb{R} \rightarrow \mathbb{R}$ is said to be regulated if it has right and left limits at each point. The space of all regulated functions of period 2π will be denoted by $P(2\pi)$. If X is a class of functions, then X_c will denote the continuous functions in class X .

The finite system of nonoverlapping intervals $\{I_i\}$ $i = 1, \dots, n$ is said to be an ordered system if I_i is to the left of I_{i+1} for all $i = 1, \dots, n-1$, or if I_i is to the right of I_{i+1} for every $i = 1, \dots, n-1$.

If $I = [a, b] \subset [-\pi, \pi]$ and $f \in P(2\pi)$, then we write $f(I) = f(b) - f(a)$. For $f \in P(2\pi)$ we will say that $f \in HBV$ if

$$\sup \sum_{i=1}^n |f(I_i)|/i = V_H(f) < \infty,$$

where the supremum is taken over all finite systems of nonoverlapping, closed intervals in $[-\pi, \pi]$. For $f \in P(2\pi)$ we will say that $f \in OHBV$ if