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Measures for Which σ-Porous Sets are Null

A set $E \subseteq R$ is said to be porous at a point x if

$$p(E,x) = \limsup \rho(x,h)/h > 0$$

where $\rho(x,h)$ is the length of the longest subinterval of $(x-h,x+h) \cap E^c$. The set E is a porous set if E is porous at each of its points, and E is σ-porous if it is the denumerable union of porous sets. Although it is evident that every σ-porous set is both of measure zero and of the first Baire category, the reverse implication is not true ([HT] or [Z]), and in applications it is often the geometry of porous sets which carries the important information. In recent years, the notion of porosity has played an important role in characterizing sets of exceptional derivative or derivate behavior and in many instances has been used to improve older results which use sets of Lebesgue measure zero which are also of the first Baire category. An interesting survey of these results can be found in B.S. Thomsons work, [TH]. One advantage to using measure is that a great deal can be brought to bear on a problem in the context of measure theory. It is natural, then, to try to define a nontrivial Borel measure on a given set in such a way that the σ-porous subsets are necessarily null. This would enable one to use the associated measure theory to study exceptional behavior. In [Tk], J. Tkadlec considered this problem and constructed a certain perfect set E of Lebesgue measure zero such that if μ is any nontrivial Borel measure on E, then there are porous subsets of E which have positive μ-measure. We call a Borel