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Intersections of Continuous Functions with Families of Smooth Functions

This paper is a report of results from joint work with A. M. Bruckner, M. Laczkovich and D. Preiss. The proofs are contained in a paper which has been submitted to TAMS. Some open questions are also included.

It is clear that for $f \in C[0,1]$, f is concave or convex on $[0,1]$ if and only if $\text{card}\,\{x : f(x) = l(x)\} \leq 2$ for all lines $l(x)$. M. Laczkovich posed the following question: If $f \in C[0,1]$ and $\{f = l\}$ is finite for all lines l, then must there be a subinterval of $[0,1]$ on which f is either concave or convex? An affirmative answer was established, which for reference we state here.

Theorem 1. If $f \in C[0,1]$ and $\{f = l\}$ is finite for all lines l, then there exists a subinterval of $[0,1]$ on which f is either concave or convex.

It is then natural to ask what results one might obtain by considering the cardinality of $\{f = l\}$ for fixed $f \in C[0,1]$ and all lines l. The following were obtained:

Theorem 2. If $f \in C[0,1]$ and $\text{card}\,\{f = l\} \leq 3$ for all lines l, then $[0,1]$ can be decomposed into 5 subintervals on each of which f is concave or convex.