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On Absolute Peano Derivatives

1. Introduction

In the recent survey article on Peano derivatives [1], M. J. Evans and C. E. Weil have stated that it is not known whether Laczkovich's absolute Peano derivatives have the -M,M property, the Zahorski property, or property Z. With some further observations on the generalized Peano derivatives studied by the author [3], we show that, in particular, the absolute Peano derivatives do have those properties.

Terminology and notations are those used in the survey article [1] unless otherwise stated. The letter n will be a positive integer throughout the paper.

Now, we review the study in [3]. The (ordinary) n^{th} Peano derivative of f at t is denoted as $f_{(n)}(t)$, the same as that in [1] except that the parentheses are put around n. The generalized n^{th} Peano derivative of f at t as defined in [3], denoted as $f_{[n]}(t)$ with brackets around n, is just the ordinary $(n+k)^{th}$ Peano derivative $g_{(n+k)}(t)$, where g is a k^{th} primitive of f in a neighborhood of t, assuming that f is continuous in that neighborhood and that there exist such k and g for which $g_{(n+k)}(t)$ exists. Note that $f_{[n]}(t)$, if it exists, is unambiguously defined since it is independent of the k and g above. Also note that it might happen that one of $f_{(n)}(t)$ and $f_{[n]}(t)$ exists while the other does not exist. However, if f is assumed to be continuous in a whole neighborhood of t, then the existence of $f_{(n)}(t)$ implies that $f_{[n]}(t)$ exists and equals $f_{(n)}(t)$. In particular, if $f_{(n)}$ exists and $f_{(1)}$ is finite in an interval, then $f_{[n]}$ exists and equals $f_{(n)}$ in that interval. The above statement fails to hold