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Two connected topologies on the real line

In this note we give answers to the following problems:

(A) Let T be a topology on the real line \mathbb{R} satisfying the following conditions:

(a) any interval is T -connected

(b) any continuous function is T -continuous

(c) if $x \in \mathbb{R}$ and U is any T -neighbourhood of x ,
then $\text{cl}U$ contains some neighbourhood of x

(d) if U is T -open and $x \in \mathbb{R}$ then $U+x$ is also T -open.

Is it true that any T -continuous function is continuous?

(B) If T satisfies (a), (b), (c) and

(e) any T -continuous function is of the second class,
is it true that any T -continuous function is continuous?

The problem (A) was posed by Professor Swiatkowski at the conference held at Lodz in the autumn 1981 and the solution of the problem (B) gives an answer to the problem (3) from [3]. We show that the answer is negative in both cases.

Lemma 1: Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be an arbitrary function and let T be