Real Analysis Exchange Vol. 8 (1982-83)
Paul D. Humke, Department of Mathemaics, St. Olaf College, Northfield,...
Minnesota.

Some Remarks on σ-Porous Sets and Unilateral Derivates

The work outlined here represents joint work with Mike Evans, Krishna Garg, Ted Vessey, and I even did a bit myself. I'll motivate this area of effort with a problem presented by Krishna Garg at the 1976 Real Analysis Conference held at Syracuse, New York. The problem is simply to characterize the following set.

$$U(f) = \{x: D^+f(x) \neq D^-f(x) \text{ or } D_+f(x) \neq D_-f(x)\}$$

This is, of course, the set of points where unilateral derivates differ, and the situation which was known at the time was:

$f \varepsilon BV$ => $U(f)$ is $G_{\delta\sigma}$, first category, measure zero.

$f \varepsilon C$ => $U(f) = U_1 \cup U_2$ where U_1 is as above, and U_2 is an arbitrary G_σ.

Since 1976 a bit more has been determined and the situation now is:

(1) $f \varepsilon L$ (Lipschitz) => $U(f)$ is $G_{\delta\sigma}$, σ-porous

(2) $f \varepsilon BV$ => $U(f)$ is $G_{\delta\sigma}$, first category, measure zero, but not necessarily σ-porous.

I'd like to discuss the converse of each of (1) and (2), and I'll begin with (2), and in particular, the following useful lemma of Zahorski.

LEMMA Z. <u>To every linear G_δ set E which does not contain an interval there corresponds a decreasing sequence of open sets G_n with</u>