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Analytic Capacity and Differentiability Properties of Finely Harmonic Functions

Summary

Let f be a finely harmonic function defined in a finely open set V in the complex plane. We investigate two kinds of differentiability properties of f, and prove the following:

1) If g_n are harmonic functions in a neighbourhood of $\bar{V}$ converging uniformly on V to f, $|g_n - f| < 2^{-n}$, then ∇g_n converge (to a limit depending only on f) everywhere outside a set of Newtonian capacity zero, and almost everywhere (wrt. arc length) on any rectifiable arc. On the other hand, ∇g_n may diverge everywhere on a given set of zero <u>analytic</u> capacity.

2) The proofs apply to give an estimate for analytic capacity. This estimate in turn implies that any compact set of Hausdorff dimension less than 1 is γ-negligible, i.e. negligible wrt. approximation by bounded analytic functions.

3) If E is a compact set of zero capacity wrt. the kernel $h(z) = -|z|^{-1}\log|z|$, then there exists a finely harmonic function f on a finely open set $V \supseteq E$ such that f is not <u>finely differentiable</u> at any point of E. This is the converse of a result due to Fuglede and Mizuta.

Details will appear in Acta Mathematica.