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Jan Mařík, Department of Mathematics, Michigan State University, East Lansing, Michigan 48824.

GENERALIZED DERIVATIVES

The classical assertion that there exist continuous, nowhere differentiable functions can be generalized in various ways. One such possibility was shown by L. Filipczak in [1]. He constructed a periodic continuous function whose upper and lower symmetric derivates are ∞ and $-\infty$, respectively, at each point. I would like to mention some theorems of J.C. Georgiou and myself that together generalize Filipczak's result.

Let r be a natural number and let $a_0 < a_1 < \ldots < a_r$. There are b_j such that $\sum_{j=0}^{r} b_j a_j^k = 0$ for $k = 0,1,\ldots, r-1$ and $\sum_{j=0}^{r} b_j a_j^r = r!$. For each finite real function f on $R = (-\infty,\infty)$ and each pair of real numbers x,h with $h \neq 0$ we define $L(f,x,h) = \sum_{j=0}^{r} b_j f(x+a_j h)$, $\lambda(f,x,h) = h^{-r}\cdot L(f,x,h)$. It is easy to see that $\lambda(f,x,h) \to f_{(r)}(x)$ $(h \to 0)$, if the r-th Peano derivative $f_{(r)}(x)$ exists. If $a_j = j - \frac{r}{2}$ for $j = 0,\ldots,r$, then $\lim \lambda(f,x,h)$ means the r-th Riemann derivative of f at x.

Now we may ask whether there is an f with the following property: