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Having Small Selectors

A linear set S will be called a selector with respect to translations for a linear E if every translate E + x has non-empty intersection with S. Equivalent forms of that relationship between S and E are: the set of distances between x and y, where $x \in E$ and $y \in S$ forms the set of all positive numbers; or: the product $E \times S$ when projected in $\mathbb{R}^2$ -plane along one of the two diagonals on one of the coordinate-axis, covers the whole axis (or, using a more picturesque form of speech, $E \times S$ is "opaque" to a bunch of parallel rays running along one of the diagonals).

My talk is about having small selectors. Let me give the general idea what it is about. Suppose that there is given some class $\mathbb{E}$ of subsets of $\mathbb{R}^1$ and we wish for any set from that class to have a selector with respect to translations, which would have certain desirable properties. For instance, we may wish that selector to be "small" in a certain sense which is intuitive and definable in terms of structural properties of $\mathbb{R}^1$ (for instance, in topological terms or in terms of measure). The question is: could we have for each set from $\mathbb{E}$ a selector which would be "small"? That question becomes interesting, when the class $\mathbb{E}$ is sufficiently large but is not restricted just to "large" sets (again, "largeness" must be understood in an appropriate sense). To be asking such and similar questions and answering them if possible, is the guiding idea of these investigations, that I am going to talk about.