QUERIES

38. *Proposed by G. Brauer, University of Minnesota*

Let $f(x)$ be a continuous real-valued function defined on the interval $I=[0,1]$ such that $f(0)=0$, $0\leq f(x)<x$ for $x>0$. Let the series $\sum_{n=0}^{\infty} u_n$ be formed from the function $f(x)$ by defining $u_0=x$, $u_{n+1}=f(u_n)$ for $n\geq 0$. If the series $\sum u_n$ converges, x is called a point of convergence; otherwise x is called a point of divergence. The set of points of convergence is called the set of convergence. It is shown (G. Brauer, Sets of convergence for series defined by iteration, Canada. Math. Bull. 9 (1966) 83-87) that the set of convergence is of type F_σ. Suppose C is an F_σ subset of I containing 0 which can be expressed as the union of a finite or infinite number of pairwise disjoint closed intervals (some of which may reduce to points) such that if $D=I\setminus C$ is non-empty, then (i) there are points of D between each pair of the closed intervals whose union is C and (ii) 0 is in the closure of D. Is there a function f satisfying the conditions stated above, such that the set of convergence is precisely C? Fort and Schuster have shown (Convergence of series whose terms are defined by iteration, Amer. Math. Monthly 71 (1964) 994-998) that if f has a positive derivative which is bounded away from zero and $f(x_1)/x_1>f(x_2)/x_2>0$ whenever $0<x_1<x_2<1$, then the set of convergence is either $\{0\}$ or the entire unit interval.