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On a Class of Orthogonal Series

In [2], Skvorcov introduced a generalization of the Perron integral for the purpose of calculation of the coefficients of a Haar series. I would like to mention some results of J. C. Georgiou and myself which extend Skvorcov's theorems to a wider class of orthogonal series. Some related questions have been studied, e.g., in [4] and [5].

1. Let V be a real vector space and let S be a subspace of V. Suppose that φ is a function on $S \times V$ such that $\varphi(s,.)$ is linear on V for each $s \in S$, $\varphi(.,v)$ is linear on S for each $v \in V$, $\varphi(s,s) > 0$ for each $s \in S \setminus \{0\}$ and that $\varphi(s,v) = \varphi(v,s)$, whenever $s, v \in S$. The restriction of φ to $S \times S$ is, obviously, an inner product so that we may speak about orthogonality in S.

Let T be a finite-dimensional subspace of S and let $v \in V$. It is easy to see that there is a unique $p \in T$ such that $\varphi(t,v) = \varphi(t,p)$ for each $t \in T$; write $p = \text{o.p.}(v,T)$ (orthogonal projection of v to T). If $T_0, T_1, \ldots$ are pairwise orthogonal finite-dimensional subspaces of S and if $v \in V$, then $\sum_{n=0}^{\infty} \text{o.p.}(v,T_n)$ will be