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Clifford E. Weil, Department of Mathematics, Michigan State University, East Lansing, Michigan, 48824, and Richard J. O'Malley, Department of Mathematics, University of Wisconsin, Milwaukee, Wisconsin 53201.

Iterated L^p Derivatives

If a function f has a k-th derivative in the usual sense at x, then according to the classical Taylor Theorem,

$$f(x+t) - f(x) - t f'(x) - \cdots - (t^k/k!) f^{(k)}(x) = o(t^k)$$

as t tends to 0. When such a formula holds (which is possible without $f^{(k)}(x)$ existing) f is said to have a k-th Peano derivative. More specifically f has a k-th Peano derivative at x if there are k-numbers, $f_1(x), \cdots, f_k(x)$ such that

$$f(x+t) - f(x) - t f_1(x) - \cdots - (t^k/k!) f_k(x) = o(t^k)$$

as t tends to 0. Interpret this condition as saying that the supremum of the left hand side over t between 0 and h; that is, the L^∞-norm of the left hand side on the interval between 0 and h, is $o(h^k)$ as h tends to 0. It is natural to replace L^∞-norm by L^p-norm, but it must be normalized so that the function identically 1 has L^p-norm 1. When this is done we have the definition of the k-th derivative in L^p. Precisely, f is said to have a k-th derivative in L^p, $0 < p < \infty$, at x if there are numbers $f_{p,1}(x), \cdots, f_{p,k}(x)$ such that