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Symmetric and Ordinary Differentiation

Let f be a real valued function defined on the real line R. The upper symmetric derivate of f at $x \in R$ is

$$D^s f(x) = \limsup_{h \to 0} \frac{f(x+h)-f(x-h)}{2h},$$

and the lower symmetric derivate $D_s f(x)$ of f at x is defined analogously. If $D^s f(x) = D_s f(x)$, then the common value is called the symmetric derivative of f at x, denoted by $f^s(x)$, and in this case we say that f is symmetrically differentiable at x.

In 1927 A. Khintchine [5] proved: a measurable function f: $R \to R$ has a finite ordinary derivative at almost every point x where $D_s f(x) > -\infty$. That the measure zero exceptional set cannot be replaced by a first category set (even for a continuous function) is demonstrated by the function constructed by Z. Zahorski in Lemma III of [8]. (See also Lemma 2 in [7]).

However, our interest here is the improvement of the following interesting consequence of Khintchine's result.