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Differentiation and Lusin's Condition (N)

This paper deals with a problem mentioned by Professor D.W. Solomon; namely, whether two continuous functions can each satisfy Lusin's condition (N), be differentiable a.e. with identical derivatives a.e. and not differ from each other by a constant. That this can occur is shown in the example below. The functions in the example differ by a monotone function and Theorem 1 shows that a function which has a pair of this type also has a pair which differs from it by a monotone function. Theorem 2 shows that no function with a pair can be ACG .

Example: There exist two continuous functions f_1 and f_2 which satisfy Lusin's condition (N), are differentiable a.e. with equal derivatives a.e., such that $f_1 - f_2$ is not identically constant.

Proof: Note that each real number $x \in [0,1]$ can be written as $\Sigma x_i \cdot 16^{-i}$ where $0 \leq x_i < 16$ or, alternatively, as $\Sigma(\frac{1}{2}x_i) \cdot 8^{-i}$ where $0 \leq x_i < 16$ and each x_i is even. Let P be the set of all $x = \Sigma x_i \cdot 16^{-i}$ where $0 \leq x_i < 16$ and each x_i is even. Then P is perfect, of measure 0, and contained in [0,15/16]. If $x \in P$ and $x = \Sigma x_i \cdot 16^{-i}$, define $f_1(x)$ by $f_1(x) = \Sigma a_i \cdot 8^{-i}$