TOPICAL SURVEY *Real Analysis Exchange Vol. 3 (1977-78)*

Lebesgue Equivalence

by

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We consider real functions on a closed interval [a,b]. Functions f and g are said to be Lebesgue equivalent if there is a homeomorphism h of [a,b] onto itself such that $g = f \circ h$, Lebesgue equivalence clearly satisfies the conditions of an equivalence relation. We shall be concerned with two sorts of questions:

(i) Does a given equivalence class contain a "nice" function?

(ii) Are all functions in a given equivalence class well behaved?

A related notion is that of Lebesgue equivalence of sets. Two sets S and T, contained in [a,b], are said to be Lebesgue equivalent if there is a homeomorphism h of [a,b] onto itself such that $T = h(S)$.

We shall discuss only matters of special interest to us. Some of these questions are related to functions whose Fourier series converge everywhere.

1. Our first remarks pertain to the well known theorem of Maximov, [13]. It is an elementary fact that every derivative is of class Baire 1 and has the