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Z. Grande, Instytut Matematyki WSP, ul. Arciszewskiego 22b, 76–200 Słupsk, Poland

T. Natkaniec, Instytut Matematyki WSP, ul. Chodkiewicza 30, 85–064 Bydgoszcz, Poland

On Some Topologies of O'Malley's Type on the Plane

Let $(X, \mathcal{T})$ be a topological space. A real function f defined on X is said to be

- $\mathcal{T}$-quasi-continuous at a point $x_0 \in X$ iff for every $\varepsilon > 0$ and for any neighbourhood $U \in \mathcal{T}$ of the point x_0 there exists $V \in \mathcal{T}$ such that $\emptyset \neq V \subset U$ and $|f(x) - f(x_0)| < \varepsilon$ for every $x \in V$,
- $\mathcal{T}$-cliquish at $x_0 \in X$ iff for every $\varepsilon > 0$ and for any neighbourhood $U \in \mathcal{T}$ of the point x_0 there exists $V \in \mathcal{T}$ such that $\emptyset \neq V \subset U$ and $osc_V f < \varepsilon$.

If $\mathcal{T}$ is the Euclidean topology on $\mathbb{R}^n$, we will write "quasi-continuous", "cliquish" instead of "$\mathcal{T}$-quasi-continuous" and "$\mathcal{T}$-cliquish".

In the present paper we study the families of $\mathcal{T}$-quasi-continuous functions and $\mathcal{T}$-cliquish functions defined on $\mathbb{R}^2$ with some topologies of density type.

I. S. Kempisty proved in [8] that if every x-section of $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$, $f_x(t) = f(x,t)$ and every y-section of f, $f^y(t) = f(t,y)$ is quasi-continuous then f is quasi-continuous, too. Note that the analogous theorem is not true for density topology d (see e.g. [2], p. 20, for definitions).

Example 1 *Under Martin's Axiom there exists a function $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ such that*

1. *all f_x and f^y sections of f are d-quasi-continuous,*
2. *f is not $d \times d$-cliquish (thus f is not $d \times d$-quasi-continuous),*
3. *f is not measurable [6]*

The following Lemma is proved in [5], p. 13.

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