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Quasi-Continuous and Cliquish Selections of Multifunctions on Product Spaces

The notions of the quasi-continuity and cliquishness have been intensively studied for their close relation to continuity. The survey papers [2], [8], [11] contain among other things also results concerning separate and joint quasi-continuity as well as the fundamental results of the set of all continuity points of quasi-continuous functions and multifunctions. Studying these problems it is not possible to avoid other generalized continuity notions such as cliquishness, pointwise discontinuity and $\mathcal{B}$-continuity. From among the papers of this kind let us mention e.g. [1], [6], [7], [12]. Many results which are valid for functions may be transferred to multifunctions. The main aim of the present paper is to extend for multifunctions some known notions and techniques about cliquishness, pointwise discontinuity and quasi-continuity of functions defined on product spaces.

In what follows X, Y are topological spaces and M metric one. A multifunction $F : X \to \mathcal{K}(Y)$ is a set valued function which assigns to each element x of X a set $F(x) \in \mathcal{K}(Y) = \{A \subset Y : A \text{ is non-empty compact }\}$. A selection of F is any function $f : X \to Y$ such that $f(x) \in F(x)$ for any $x \in X$. If F is a multifunction defined on the product space $X \times Y$ we shall call an x-section for given $x \in X$ the multifunction $F_x(y) = F(x, y)$. The y-section F_y for a given $y \in Y$ is defined analogously

The upper (lower) inverse image $F^+(A)$ $(F^-(A))$ is defined for any set $A \subset Y$ as

$$F^+(A) = \{x \in X : F(x) \subset A\}, \quad F^-(A) = \{x \in X : F(x) \cap A \neq \emptyset\}$$

By $S_\varepsilon(A)$ we denote an ε-neighborhood of $A \subset M$, $\varepsilon > 0$ i.e., $S_\varepsilon(A) = \{z \in M : d(z, A) < \varepsilon\}$ where d is a metric for M. If $A = \{y\}$, we briefly write $S_\varepsilon(y)$. By $\text{int}(B)$ we denote the interior of B.

Definition 1 *A multifunction $F : X \to \mathcal{K}(M)$ is said to be cliquish at a point $p \in X$ if for any $\varepsilon > 0$ and any neighborhood U of p there is a non-empty open*

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